

Problem Set 5

Textbook questions are from “Galois Theory” (2nd edition) by Joseph Rotman.

1. Textbook exercise 63.
2. Textbook exercise 66.
3. Show that $x^4 + 1$ is irreducible over $\mathbb{Q}[x]$ but not over $\mathbb{R}[x]$.
4. Let $f(x) = x^3 + x^2 + x + 2$.
 - (a) Is $f(x)$ irreducible over $\mathbb{Z}_5[x]$? Just from this answer can you conclude anything about irreducibility of $f(x)$ over $\mathbb{Q}[x]$?
 - (b) Is $f(x)$ irreducible over $\mathbb{Z}_3[x]$? Just from this answer can you conclude anything about irreducibility of $f(x)$ over $\mathbb{Q}[x]$?
5. Let p be a prime of the form $4k + 3$ for some $k \in \mathbb{Z}_{\geq 0}$.
Show that $x^2 + 1$ is irreducible in $\mathbb{Z}_p[x]$.

Hint : multiplicative order of a root.
